10.61. Solve: (a) The equilibrium positions are located at points where $\frac{dU}{dx} = 0$.

$$\frac{dU}{dx} = 0 = 1 + 2\cos(2x) \Rightarrow \cos(2x) = -\frac{1}{2}$$
$$\Rightarrow x = \frac{1}{2}\cos^{-1}\left(-\frac{1}{2}\right)$$

Note that $-\frac{1}{2}$ is in radians and x is in meters. The function $\cos^{-1}\left(-\frac{1}{2}\right)$ may have values $\frac{2\pi}{3}$ and $\frac{4\pi}{3}$. Thus there are two values of x,

$$x_1 = \frac{\pi}{3} \text{ and } x_2 = \frac{2\pi}{3}$$

within the interval $0 \text{ m} \le x \le \pi \text{ m}$.

(b) A point of stable equilibrium corresponds to a local minimum, while a point of unstable equilibrium corresponds to a local maximum. Compute the concavity of $U(x)$ at the equilibrium positions to determine their stability.

$$\frac{d^2U}{dx^2} = -4\sin(2x)$$

At $x_1 = \frac{\pi}{3}$, $\frac{d^2U}{dx^2}(x_1) = -4\left(\frac{\sqrt{3}}{2}\right) = -2\sqrt{3}$. Since $\frac{d^2U}{dx^2}(x_1) < 0$, $x_1 = \frac{\pi}{3}$ is a local maximum, so $x_1 = \frac{\pi}{3}$ is a point of unstable equilibrium.

At $x_2 = \frac{2\pi}{3}$, $\frac{d^2U}{dx^2}(-x_2) = -4\left(-\frac{\sqrt{3}}{2}\right) = +2\sqrt{3}$. Since $\frac{d^2U}{dx^2} > 0$, $x_2 = \frac{2\pi}{3}$ is a local minimum, so $x_2 = \frac{2\pi}{3}$ is a point of stable equilibrium.